**Teacher Notes**

Objective: Students will be able to derive the equation of a circle based on an exploration of right triangles on the coordinate plane. Students will end up creating their own version of the Unit Circle.

Materials (per team):

* Instruction sheets
* Poster paper
* Set of triangles
* Markers

Teaming: Students should be in teams of 2-3.

**Instructions:**

1. Create a set of two right triangles on a page.
   1. Both triangles need to have the same hypotenuse.
   2. For this activity to work well on a piece of ordinary poster paper, triangles should have a hypotenuse of about 9 inches.
   3. The angles of one triangle should be 30-60-90 while the other one should be 45-45-90. The angles should be labeled.
   4. The hypotenuse should be labeled “1.”
2. Explain to students they are exploring special properties of right triangles on a coordinate plane. Students should not be told the relationship between triangles and circles -- they are going to discover this.
3. Give students the materials listed above.
4. Have students follow the instructions on the instruction sheets.
5. Circulate around the room to ensure students are properly placing triangles and are not moving ahead prematurely. Check in with students formally at the designated checkpoints.
6. When most teams are complete, answer the unit circle questions with the class and formalize through notes.

The following two pages are instructions to be printed out for students.

**Part 1: Special Right Triangles on the Coordinate Plane**

1. Cut out the triangles.



1. The hypotenuse of each triangle is 1. Find the missing side lengths, using trigonometry or special right triangles.
2. Draw a coordinate plane the size of your poster, with the origin close to the center.
3. Place a triangle on coordinate plane with a non-right angle at the origin and the adjacent side on the x-axis (so the right angle is on the x-axis).

 

1. Lightly trace the triangle onto your grid. Mark the point of the other non-right angle on your coordinate plane and label this point with its coordinate (x, y). Then, label the inside angle (angle at the origin) for this triangle on your coordinate plane.



1. Repeat steps 4-6 with both triangles in all possible configurations around the origin (hint: there should be 12!).
2. What shape do you notice is formed by your points?
3. Draw this shape on your grid.

STOP! Check with your teacher for further instructions.

**Part 2: The Unit Circle**

Congratulations! Your group has just created a famous math tool called the Unit Circle.

In your journal:

1. Write a summary of the activity so far. Your summary should be in complete sentences and be 4-6 sentences long. Include what you did in the activity and any “a-ha!” moments you had.
2. Answer the following questions using your unit circle:
   1. What is the radius of this circle?
   2. If you didn’t know the radius of the circle, how could you find the radius using one of the right triangles? Choose a right triangle and show your work.
   3. Is it possible to draw other right triangles with their hypotenuse between the origin and edge of the circle? How many right triangles are possible?
   4. What is the relationship between the legs of the right triangles and the (x, y) coordinates?
   5. How can you find the radius of the circle given any coordinates on the edge of the circle?
   6. Write an equation to express the relationship between the (x, y) coordinates of a circle and the radius of the circle.
   7. What does this equation remind you of?

STOP! Check with your teacher for further instructions.

**Extender: The Unit Circle and Trig**

Now that you know the relationship between right triangles and circles, let’s look a little more at how the unit circle is useful in trigonometry.

1. Choose a right triangle to find the sin ϴ, cos ϴ, and tan ϴ ratios, with ϴ being the angle at the origin. What is the relationship between these trig ratios and the (x, y) coordinates of this triangle?
2. What is the relationship between any triangle coordinates and the sin ϴ, cos ϴ, and tan ϴ ratios ratios?
3. Why does this relationship make sense?
4. How can the unit circle be useful when doing trigonometry?
5. Draw a copy of the unit circle in your notes for your own reference. Keep this handy for next year’s math course :)